Dynamics of the Scalar Condensate in thermal 4D self-interacting Scalar Field Theory on the Lattice

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We simulate a four dimensional self-interacting scalar field theory on the lattice at finite temperature. By varying temperature, the system undergoes a phase transition from broken phase to symmetric phase. Our data show that the zero-momentum field renormalization increases by approaching critical temperature. On the other hand, finite-momentum wave-function renormalization remains remarkably constant.

Traditionally, the 'condensation' of a scalar field, i.e. the transition from a symmetric phase where $\langle\Phi\rangle = 0$ to the physical vacuum where $\langle\Phi\rangle \neq 0$, has been described as an essentially classical phenomenon in terms of a classical potential

$$V_{\rm cl}(\Phi) = \frac{1}{2} r_0 \Phi^2 + \frac{\lambda_0}{4!}\Phi^4 \qquad (1)$$

with non-trivial absolute minima at $\Phi = \pm v_B \neq 0$ ('B=Bare'). In this picture, the 'scalar condensate' is treated as a classical c-number field which is simply taken into account through a shift of the scalar field, say $\Phi(x) = v_B + h(x)$. In this picture, by retaining terms at most quadratic in the shifted field $h(x)$, one expects a simple relation

$$M_h^2 = V''_{\rm cl}(\Phi = v_B) \qquad (2)$$

relating the 'Higgs mass' M_h directly to the quadratic shape of the potential at the minimum. Beyond the tree-approximation, and on the basis of perturbation theory, Eq. (2) is believed to represent a good approximation provided the classical potential is replaced with the full quantum effective potential $V_{\rm eff}$.

However, there is [1,2] an alternative non-perturbative description of spontaneous symmetry breaking in which the zero-momentum limit is not smooth in the broken phase, reflecting the discontinuity inherent in a Bose-Einstein condensation phenomenon, where a single mode acquires a macroscopic occupation number. In this picture, M_h^2 and the curvature of the effective potential at the non-trivial minima are *different* physical quantities related by an infinite renormalization in the continuum limit of quantum field theory.

To test this prediction objectively one can proceed as follows. Let the zero-momentum two-point function (the inverse susceptibility) be related to the Higgs mass through a re-scaling factor $Z = Z_\varphi$ *defined* as

$$\chi^{-1} = \Gamma_2(p=0) = \left.\frac{d^2 V_{\rm eff}}{d\varphi_B^2}\right|_{\varphi_B = \pm v_B} \equiv \frac{M_h^2}{Z_\varphi}\,. \qquad (3)$$

This definition of $Z = Z_\varphi$ is, *a priori*, different from the quantity $Z = Z_{\rm prop}$ defined from a single-pole form of the propagator

$$G_{\rm pole}(p) = \frac{Z_{\rm prop}}{p^2 + m^2_{\rm prop}}\,. \qquad (4)$$

Indeed, the two quantities agree only if Eq. (4) remains valid down to $p_\mu = 0$ (up to small perturbative corrections).

In the continuum limit the alternative description of SSB predicts that $Z_\varphi \to \infty$ but $Z_{\rm prop} \to 1$.

In this respect, this picture is very different from renormalized perturbation theory where the zero-momentum limit is smooth and the continuum limit corresponds to

$$Z_\varphi|_{\rm pert} = Z_{\rm \ prop}[1 + \mathcal{O}(\lambda_R)]\,. \quad (5)$$

To measure on the lattice Z_φ one has to measure $\Gamma_2(0)$ and M_h. While the zero-momentum two-point function is directly obtained from the inverse susceptibility χ^{-1}, determining M_h requires to study the lattice propagator. This will be described below. Our numerical simulations were performed (using the Swendsen-Wang cluster algorithm) in the Ising limit of the theory where a one-component $(\lambda\Phi^4)_4$ theory becomes

$$S = -\kappa \sum_{x,\mu} [\phi(x+\hat{\mu})\phi(x) + \phi(x-\hat{\mu})\phi(x)] \quad (6)$$

with $\Phi(x) = \sqrt{2\kappa}\phi(x)$ and where $\phi(x)$ takes the values ± 1. We measured the bare magnetization

$$v_B = \langle|\Phi|\rangle\,, \Phi \equiv \sum_x \Phi(x)/L^4\,, \quad (7)$$

the zero-momentum susceptibility

$$\chi = L^4 \left[\left\langle|\Phi|^2\right\rangle - \langle|\Phi|\rangle^2\right]\,, \quad (8)$$

and the shifted-field propagator

$$G(p) = \sum_x \exp(ipx)\,\langle h(x)h(0)\rangle\,, \quad (9)$$

$p \equiv (p_4, \mathbf{p})$ with $\mathbf{p} = 2\pi\mathbf{n}/L_s$ and $p_4 = 2\pi n_4/L_t$ (n_1, n_2, n_3, n_4) are integer-valued components, not all zero. Zero-temperature simulations correspond to $L_t = L_s = L$ and will be discussed first.

To determine the Higgs boson mass one has to compare the lattice propagator with the (lattice version of the) single-pole form Eq. (4) by extracting $M_h = m_{\rm prop}$ from a two-parameter fit to the lattice data:

$$G^{(\beta)}_{\rm fit}(n, \mathbf{p}) = \frac{Z_{\rm prop}}{\hat{\mathbf{p}}^2 + \hat{p_4}^2 + m^2_{\rm prop}} \quad (10)$$

where $m_{\rm prop}$ is the mass in lattice units and $\hat{p}_\mu = 2\sin(p_\mu/2)$. In the symmetric phase (i.e. for values of the hopping parameter $\kappa < \kappa_c \sim 0.0748$), Eq. (10) provides a very good description of the lattice data for the propagator in the whole range of Euclidean momenta, namely from $p = 0$ up to $p^2 \sim \Lambda^2$ ($\Lambda \sim \frac{\pi}{a}$ is the ultraviolet cutoff).

In the broken phase, however, the attempt to fit all data with the same pair of parameters $(m_{\rm prop}, Z_{\rm prop})$ gives unacceptably large values of the normalized chi-squared. Still, one can obtain a good-quality fit with Eq. (10) by suitably restricting the range of momenta. In this case there are two choices: (i) a high-momentum range: $p^2_{\rm min} \leq p^2 \leq \Lambda^2$ (with $p^2_{\rm min} \neq 0$); (ii) a low-momentum range: $0 \leq p^2 \leq p^2_{\rm max}$ (with $p^2_{\rm max} \ll \Lambda^2$). If we require agreement of $Z_{\rm prop}$ with $Z_{\rm pert}$ (the normalization of massive single-particle states as computed in perturbation theory) the Higgs mass turns out to be determined from the set (i). Indeed we found [3] that the choice (i) to compute the Higgs boson mass and the normalization of single-particle states satisfies several consistency checks. As concern Z_φ introduced in Eq. (3), our previous analysis [3] shows that there is a discrepancy between Z_φ and $Z_{\rm prop}$. Moreover, this discrepancy becomes larger when $\kappa \to \kappa_c$, where the purely perturbative corrections $\mathcal{O}(\lambda_R)$ in Eq. (5) vanish. For instance, on a 32^4 lattice at $\kappa = 0.07512$, a fit to the propagator data gives [3] $M_h = m_{\rm prop} = 0.206(4)$ and $Z_{\rm prop} = 0.9551(21)$. However, the measured susceptibility is $\chi = 193.1 \pm 1.7$, so that $Z_\varphi = 2\kappa\chi m^2_{\rm prop} = 1.234(50)$, at more than 5σ's from $Z_{\rm prop}$.

The same discrepancy can be presented in a different way. To check that the discrepancy is not due to finite-size effects, we have repeated the measurement of χ for $\kappa = 0.07512$ on a 40^4 lattice with the result $\chi = 190.9 \pm 1.6$. If we now take the mass value $m_R = 0.20$ (for $\kappa = 0.0751(1)$) from Table 3 in [4] we get $Z_\varphi = 2\kappa\chi m^2_R = 1.147(10)$ at about 14σ's from the perturbative prediction $Z_{\rm pert} = 0.937(12)$ in the same Table.

To provide further evidences, we shall now present new lattice results. These suggest that the discrepancy between the zero-momentum Z_φ and its perturbative prediction $Z_{\rm pert} \sim Z_{\rm prop}$ is due to the presence of a non-vanishing scalar condensate in the broken phase. To this end, we have performed finite-temperature simulations by considering asymmetric lattice, $L_s^3 \times L_t$, periodic in time direction. As it is well known, this is equivalent to a non-zero temperature $T = 1/L_t$.

At $\kappa = 0.07512$ (and for all values of L_s) there is clear evidence for a phase transition in the region $6 < L_t < 8$ where the system crosses from the broken into the symmetric phase (see Fig. 1). Our lattice results show that, well above

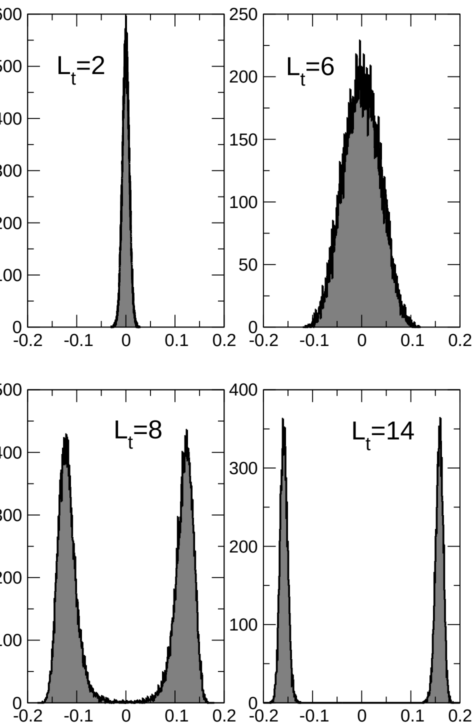

Figure 1. The distribution of the average field for $L_s = 64$ and four values of L_t.

the phase transition temperature, i.e. for very small L_t, when the system is in the symmetric phase, the data for the propagator are well reproduced by Eq. (10) down to $p = 0$. Therefore, at high temperature the zero-momentum limit is smooth so that Z_φ and $Z_{\rm prop}$ agree to very good accuracy, as they do in a $T = 0$ symmetric-phase simulation for $\kappa < \kappa_c$. Thus the broken-phase discrepancy between Z_φ and $Z_{\rm prop}$ is a real physical effect that could be ascribed to the presence of a non-vanishing scalar condensate in the broken phase. In Fig. 2 we have summarized our data for Z_φ. They show a clear increase when $T \to T_c$ in contrast with the values of $Z_{\rm prop}$ (not shown) that remain remarkably close to the zero-temperature value 0.9551. The trend for $T \to T_c$ is in qualitative agreement with our previous zero-temperature study for $\kappa \to \kappa_c$. We stress that the

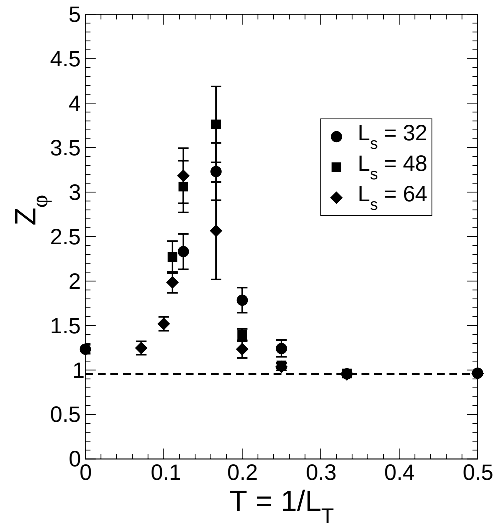

Figure 2. Lattice data for $Z_\varphi \equiv m^2_{\rm prop}\chi$ vs. T. Dashed line is the T=0 value $Z_{\rm prop} = 0.9551$.

conventional interpretation of 'triviality', assuming $Z_\varphi = Z_{\rm prop} \simeq 1$, predicts that approaching the phase transition one should find $m^2 \to 0$ and $\chi \to \infty$ in such a way that $m^2\chi$ remains constant. In this sense, the finite-temperature simulations, showing a dramatic increase of $Z_\varphi = 2\kappa\chi m^2_{\rm prop}$ when approaching the phase transition, confirm and extend the previous zero-temperature results where the continuum limit was approached by letting $\kappa \to \kappa_c$. Both show that Z_φ is very different from the more conventional quantity $Z_{\rm prop} \sim 1$ determined perturbatively from the residue of the shifted field propagator. For this reason, the lattice data support once more the alternative picture of Refs. [1,2] where an infinitesimal curvature of the effective potential $V''_{\rm eff} = M_h^2/Z_\varphi \to 0$ can be reconciled with finite values of M_h. This non-trivial result may have important consequences for particle physics and cosmology

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